(a)

$u(x) = a\,x^3 + b\,x^2 + c\,x + d$ ... die gesuchte Par. 3ter Ordnung berührt

$y(x) = \frac{1}{4}x^2$ .... in (0; 0) wenn

$u(0) = y(0) = 0$ und $u'(0) = y'(0)$ gelten; wegen $y'(x) = \frac{1}{2}x,\, y'(0) = 0$

$u(0) = 0 \Longrightarrow u(x) = a\,x^3 + b\,x^2 + c\,x,\; u'(x) = 3a\,x^2 + 2b\,x + c$

$u'(0) = 0 \Longrightarrow u(x) = a\,x^3 + b\,x^2,\;\; u'(x) = 3a\,x^2 + 2b\,x$

(I) Hochpunkt $(5; \frac{25}{49}) \Longrightarrow u(5) = 125a + 25b = \frac{25}{49}$

(II) und $\Longrightarrow u'(5) = \;\; 75a + 10b = 0 \Rightarrow b = \frac{-15}{2}a$

eingesetz in (I) $125a - \frac{375}{2}a = \frac{25}{49} = -\frac{125}{2}a$, $\boxed{a = \frac{-50}{125*49} = \frac{-2}{5*49};\, b = \frac{+3}{49}}$

$$\boxed{\boldsymbol{u(x) = \frac{-2}{5*49}x^3 + \frac{3}{49}x^2}}$$

(b) Fläche zwischen den beiden Kurven

2ter „Schnittpunkt“ $(v; v^2/4)$; [1ter ist (0; 0)]

$u(v) = y(v)$ also $\frac{-2}{5*49}v^3 + \frac{3}{49}v^2 = \frac{1}{4}v^2$, $\frac{-2}{5*49}v + \frac{3}{49} = \frac{1}{4}$; $\boldsymbol{u = -\frac{185}{8}}$

Fläche

$$A_1 = \left| \int_{-185/8}^{0} \left(\frac{-2}{5*49}x^3 + \frac{3}{49}x^2 - \frac{1}{4}x^2 \right) \mathrm{dx} \right|$$

$$A_1 = \left| \int_{185/8}^{0} \left(\frac{-2}{5*49}x^3 - \frac{137}{196}x^2 \right) \mathrm{dx} \right|$$

$$A_1 = \left| \left| \frac{-2x^4}{20*49} - \frac{137\,x^3}{3*196} \right|_{-185/8}^{0} \right| = \left| \frac{-2*(185/8)^4}{20*49} + \frac{137*(185/8)^3}{3*196} \right| \doteq 194,540$$