$$I=\int \ln(ax^2+bx+c)\mathrm{dx}=\int u\mathrm{dv}=uv-\int v\mathrm{du}$$

$$u=\ln(...),\ \mathrm{du}=\frac{2ax+b}{ax^2+bx+c}\mathrm{dx};\quad \mathrm{dv}=\mathrm{dx},\ v=x$$

$$I=x\ln(...)-\int \frac{2ax^2+bx}{ax^2+bx+c}\mathrm{dx},\ \frac{2ax^2+bx}{ax^2+bx+c}=2-\frac{bx+2c}{ax^2+bx+c}$$

$$\frac{bx+2c}{ax^2+bx+c}=\frac{b}{2a}*\frac{2ax+b}{ax^2+bx+c}-\frac{b^2}{2a}*\frac{1}{ax^2+bx+c}+\frac{2c}{ax^2+bx+c}$$

$$=\frac{b}{2a}*\frac{2ax+b}{ax^2+bx+c}+\frac{1}{2a}*\frac{4ac-b^2}{ax^2+bx+c}$$

$$I=(x+\frac{b}{2a})\ln(...)-2x+\frac{4ac-b^2}{2a}\int \frac{\mathrm{dx}}{ax^2+bx+c}$$

$$ax^2+bx+c=r^2+s=\left(x\sqrt{a}+\frac{b}{2\sqrt{a}}\right)^2+(c-\frac{b^2}{4a}),\quad \mathrm{dx}=\frac{\mathrm{dr}}{\sqrt{a}}$$

$$r^2+s=s\left((\frac{r}{\sqrt{s}})^2+1\right)=s(t^2+1);\ \mathrm{dt}=\frac{\mathrm{dr}}{\sqrt{s}},\ \mathrm{dx}=\sqrt{\frac{s}{a}}\mathrm{dt}$$

$$\int \frac{\mathrm{dx}}{ax^2+bx+c}=\sqrt{\frac{s}{a}}\int \frac{\mathrm{dt}}{s(t^2+1)}=\sqrt{\frac{1}{sa}}\mathrm{arctan}\,t$$

$$I=(x+\frac{b}{2a})\ln(...)-2x+\frac{4ac-b^2}{2a}\sqrt{\frac{1}{sa}}\mathrm{arctan}\,t;\ sa=\frac{4ac-b^2}{4}$$

$$=(x+\frac{b}{2a})\ln(...)-2x+\frac{4ac-b^2}{2a}\sqrt{\frac{4}{4ac-b^2}}\mathrm{arctan}\frac{r}{\sqrt{s}}$$

$$=(x+\frac{b}{2a})\ln(...)-2x+\frac{\sqrt{4ac-b^2}}{a}\mathrm{arctan}\frac{\frac{2ax+b}{2\sqrt{a}}}{\sqrt{\frac{4ac-b^2}{4a}}}$$

$$=\int \ln(ax^2+bx+c)\mathrm{dx}$$

$$=(x+\frac{b}{2a})\ln(ax^2+bx+c)-2x+\frac{\sqrt{4ac-b^2}}{a}\mathrm{arctan}\frac{2ax+b}{\sqrt{4ac-b^2}}$$