$$S_0 = \sum_{k=0}^{n} q^k = \frac{q^{n+1}-1}{q-1}; \; S_r = \sum_{k=1}^{n} k^r q^k; \quad r \in \mathbb{N}$$

$$S_1^* = \frac{\partial S_0}{\partial q} = \sum_{k=0}^{n} k\, q^{k-1} = \frac{(n+1)\, q^n (q-1) - (q^{n+1}-1)}{(q-1)^2} = \frac{q^{n+1}(n+1-1) - q^n (n+1) + 1}{(q-1)^2}$$

$$= \frac{n\, q^{n+1} - q^n (n+1) + 1}{(q-1)^2}; \; S_1 = \sum_{k=0}^{n} k\, q^k = \sum_{k=1}^{n} k\, q^k = q * S_1^*$$

$$\boxed{S_1 = \sum_{k=1}^{n} k\, q^k = \frac{n\, q^{n+2} - q^{n+1}(n+1) + q}{(q-1)^2}}$$

für algemeine S_r Formel besser schreiben als $\frac{q^{n+1}(qn - (n+1)) + q}{(q-1)^2}$

$$S_2^* = \frac{\partial S_1}{\partial q} = \sum_{k=1}^{n} k^2\, q^{k-1} = \frac{\left(n(n+2)\, q^{n+1} - (n+1)^2 q^n + 1\right)(q-1) - 2\left(n\, q^{n+2} - q^{n+1}(n+1) + q\right)}{(q-1)^3}$$

$$\text{Zähler} = \begin{pmatrix} q^{n+2}(n(n+2) - 2n) + \\ q^{n+1}(-(n+1)^2 - n(n+2) + 2(n+1)) + \\ q^n(+(n+1)^2) + \\ q^1(1-2) + \\ -1 \end{pmatrix} = \begin{pmatrix} q^{n+2} n^2 \\ + q^{n+1}(-2n^2 - 2n + 1) \\ + q^n (n+1)^2 \\ - q - 1 \end{pmatrix}$$

$$\boxed{S_2 = q\, S_2^* = \frac{-q(q+1) + q^{n+1}\left(q^2 n^2 + q\,(-2\,n^2 - 2\,n + 1) + (n+1)^2\right)}{(q-1)^3}}$$

Für $n \to \infty$ bleibt also $\frac{q(1+q)}{(1-q)^3}$