$$\sum_{k=0} \cos k\alpha = \mathrm{Re}/(4\sin^2\frac{\alpha}{2}),\quad \sum_{k=0} \sin k\alpha = \mathrm{Im}/(4\sin^2\frac{\alpha}{2})$$

$$\mathrm{Re} = (\cos\alpha - 1)[\cos(n+1)\alpha - 1] + \sin\alpha\sin(n+1)\alpha$$

$$\mathrm{Im} = (\cos\alpha - 1)\sin(n+1)\alpha - [\cos(n+1)\alpha - 1]\sin\alpha$$

$$\mathrm{Re} = \cos\alpha\cos(n+1)\alpha + \sin\alpha\sin(n+1)\alpha - \cos(n+1)\alpha + (1-\cos\alpha)$$

$$= \cos[(n+1)\alpha - \alpha] - \cos(n+1)\alpha + 2\sin^2\frac{\alpha}{2}$$

$$= \cos n\alpha - \cos(n+1)\alpha + 2\sin^2\frac{\alpha}{2}$$

$$= -2\sin(n\alpha + \frac{\alpha}{2})\sin\frac{-\alpha}{2} + 2\sin^2\frac{\alpha}{2}$$

$$\mathrm{Re} = 2\sin\frac{\alpha}{2}\sin(n\alpha + \frac{\alpha}{2}) + 2\sin^2\frac{\alpha}{2}$$

$$\mathrm{Re}/4\sin^2\frac{\alpha}{2} = \sum_{k=0}^{n} \cos k\alpha = \frac{\sin(n\alpha + \frac{\alpha}{2})}{2\sin\frac{\alpha}{2}} + \frac{1}{2}$$

$$\mathrm{Im} = (\cos\alpha - 1)\sin(n+1)\alpha - [\cos(n+1)\alpha - 1]\sin\alpha$$

$$\mathrm{Im} = \sin(n+1)\alpha\cos\alpha - \sin\alpha\cos(n+1)\alpha - \sin(n+1)\alpha + \sin\alpha$$

$$= \sin[(n+1)\alpha - \alpha] - \sin(n+1)\alpha + \sin\alpha$$

$$= \sin n\alpha + \sin[-(n+1)\alpha] + \sin\alpha$$

$$= 2\sin\frac{n\alpha - (n+1)\alpha}{2}\cos(n\alpha + \frac{\alpha}{2}) + \sin\alpha$$

$$= 2\sin\frac{-\alpha}{2}\cos(n\alpha + \frac{\alpha}{2}) + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$= 2\sin\frac{\alpha}{2}\,[\cos\frac{\alpha}{2} - \cos(n\alpha + \frac{\alpha}{2})]$$

$$= 2\sin\frac{\alpha}{2}\,(-2)\sin\frac{n+1}{2}\alpha\sin\frac{-n\alpha}{2}$$

$$\mathrm{Im} = 4\sin\frac{\alpha}{2}\sin\frac{n\alpha}{2}\sin\frac{n+1}{2}\alpha$$

$$\mathrm{Im}/4\sin^2\frac{\alpha}{2} = \sum_{k=0}^{n} \sin k\alpha = \frac{\sin\frac{n\alpha}{2}\sin\frac{n+1}{2}\alpha}{\sin\frac{\alpha}{2}}$$