$1a$

$\sqrt[3]{5}*7=\sqrt[3]{5}*\sqrt[3]{7^3}=\sqrt[3]{5*8^3}$

$1b$

$\sqrt[a]{x^b}*x^b=\sqrt[a]{x^b}*\sqrt[a]{x^{ab}}=\sqrt[a]{x^bx^{ab}}=\sqrt[a]{x^{b(1+a)}}$

$1c$

$(a-b)\sqrt{(a-b)}=\sqrt{(a-b)^3}$

$1d$

$(z-y)^{\frac{2}{3}}*\sqrt[3]{(y+z)^2}=\sqrt[3]{\left[(z-y)^{\frac{2}{3}}\right]^3}*\sqrt[3]{(y+z)^2}$

$=\sqrt[3]{(z-y)^2}*\sqrt[3]{(y+z)^2}=\sqrt[3]{(z-y)^2(y+z)^2}$

$=\sqrt[3]{(z^2-y^2)^2}$

$2a$

$\frac{x^n-x^{n+2}}{x^n+x^{n-1}}=\frac{x^{n-1}(x-x^3)}{x^{n-1}(x+1)}=\frac{x(1-x^2)}{1+x}=\frac{x(1+x)(1-x)}{1+x}$

$=x(1-x)$

$2b$

$\frac{t^{5x}-t^{3x}}{t^{3x+1}-t^{2x+1}}=\frac{t^{2x+1}(t^{3x-1}-t^{x-1})}{t^{2x+1}(t^x-1)}=\frac{t^{x-1}(t^{2x}-1)}{t^x-1}$

DEINES STIMMT AUCH

$2c, d$: OK

$3a$: 1ten Bruch mit x^k erweitern

$\frac{x^{-k}-1}{1-x^k}-\frac{1}{x^k}=\frac{1-x^k}{x^k(1-x^k)}-\frac{1}{x^k}=\frac{(1-x^k)-(1-x^k)}{x^k(1-x^k)}=0$

$3b$: $a^2-b^2=(a+b)(a-b)$

$\left(x^n-\frac{1}{x^n}\right)^2-x^{-2n}=\left[\left(x^n-\frac{1}{x^n}\right)+\frac{1}{x^n}\right]\left[\left(x^n-\frac{1}{x^n}\right)-\frac{1}{x^n}\right]$

$=x^n\left(x^n-\frac{2}{x^n}\right)=x^{2n}-2$

$3c$

$\frac{1-a^p}{1+a^p}-\frac{1-a^p}{3-a^p}+2=\frac{(1-a^p)((3-a^p)-(1+a^p))}{(1+a^p)(3-a^p)}+2$

$=\frac{(1-a^p)(2-2a^p)}{(1+a^p)(3-a^p)}+2=2\left(\frac{(1-a^p)^2}{(1+a^p)(3-a^p)}+1\right)$

$=2\frac{(1-a^p)^2+(3+2a^p-a^{2p})}{(1+a^p)(3-a^p)}=2\frac{4}{(1+a^p)(3-a^p)}$

$$= \frac{8}{(1+a^p)(3-a^p)}$$

$3d$: da hilft nur, ersmal auf gemeinsamen Nenner bringen

$$\frac{t^n}{t^n-2} - 1 + \frac{t^2}{t^{n-1}-2} = \frac{t^n(t^{n-1}-2) - (t^n-2)(t^{n-1}-2) + t^2(t^n-2)}{(t^n-2)(t^{n-1}-2)}$$

$$= \frac{t^{2n-1}(1-1) + t^n(-2+2) + t^2(t^n-2)}{(t^n-2)(t^{n-1}-2)} = \frac{t^2}{t^{n-1}-2}$$